Cambridge International Examinations
Cambridge Pre-U Certificate

MATHEMATICS (PRINCIPAL) **9794/02**

Paper 2 Pure Mathematics 2 **May/June 2016**

2 hours

2764008273

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF20)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

The syllabus is approved for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

This document consists of **3** printed pages and **1** blank page.

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1 **(i)** Find the remainder when $x^3 + 2x$ is divided by $x + 2$. [2]

(ii) Write down the value of k for which $x + 2$ is a factor of $x^3 + 2x + k$. [1]

2 Solve the equation $4 \times 3^x = 5$, giving the solution in an exact form. [4]

3 The graph of $\log_{10} y$ against x is a straight line with gradient 2 and the intercept on the vertical axis at 4.

Write down an equation for this straight line and show that $y = 10\,000 \times 100^x$. [4]

4 The complex numbers z_1 and z_2 are given by $z_1 = 2 + \mathrm{i}$ and $z_2 = 3 + 4\mathrm{i}$.

(i) Verify that $|z_1| + |z_2| > |z_1 + z_2|$. [4]

(ii) Sketch on an Argand diagram the locus $|z - z_1| = 2$. [2]

5 **(i)** Show that $\dfrac{3}{x+2} + \dfrac{1}{x+1} \equiv \dfrac{4x+5}{x^2+3x+2}$. [2]

(ii) Differentiate $\dfrac{4x+5}{x^2+3x+2}$ with respect to x. [3]

(iii) Hence show that the function given by

$$\mathrm{f}(x) = \frac{4x+5}{x^2+3x+2}, \quad x \neq -1,\ x \neq -2,$$

is a decreasing function. [2]

6 The points A and B are at (2, 3, 5) and (8, 2, 4) with respect to the origin O.

(i) Find the size of angle AOB. [4]

(ii) Show that triangle AOB is isosceles. [3]

7 **(i)** Use a change of sign to verify that the equation $\cos x - x = 0$ has a root α between $x = 0.7$ and $x = 0.8$. [2]

(ii) Sketch, on a single diagram, the curve $y = \cos x$ and the line $y = x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, giving the coordinates of all points of intersection with the coordinate axes. [2]

An iteration of the form $x_{n+1} = \cos(x_n)$ is to be used to find α.

(iii) By considering the gradient of $y = \cos x$, show that this iteration will converge. [3]

(iv) On a copy of your sketch from part **(ii)**, illustrate how this iteration converges to α. [2]

(v) Use a change of sign to verify that $\alpha = 0.7391$ to 4 decimal places. [2]

8 P and Q are points on the circumference of a circle with centre O and radius r. The angle POQ is θ radians. Given that the chord PQ has length 4, find an expression for the length of the arc PQ in terms of θ only. [5]

9 **(i)** Show that $\dfrac{\sin x}{1+\sin x} \equiv \sec x \tan x - \sec^2 x + 1$. [5]

(ii) Hence show that $\displaystyle\int_0^{\frac{1}{4}\pi} \frac{\sin x}{1+\sin x}\,\mathrm{d}x = \tfrac{1}{4}\pi + \sqrt{2} - 2$. [6]

10 **(i)** Using the substitution $u = \dfrac{1}{x}$, or otherwise, find $\displaystyle\int \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,\mathrm{d}x$. [4]

(ii) Evaluate $\displaystyle\int_{\frac{1}{2\pi}}^{\frac{1}{\pi}} \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,\mathrm{d}x$ and $\displaystyle\int_{\frac{1}{3\pi}}^{\frac{1}{2\pi}} \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,\mathrm{d}x$. [3]

(iii) Show that, when n is a positive integer, the integral $\displaystyle\int_{\frac{1}{(n+1)\pi}}^{\frac{1}{n\pi}} \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,\mathrm{d}x$ takes one of the two values found in part **(ii)**, distinguishing between the two cases. [3]

11 The function f is defined by $\mathrm{f}(x) = \sqrt{x}$, $x > 0$.

(a) Use differentiation from first principles to find an expression for $\mathrm{f}'(x)$. [5]

The lines l_1 and l_2 are the tangents to the curve $y = \mathrm{f}(x)$ at the points A and B where $x = a$ and $x = b$ respectively, $a \neq b$.

(b) **(i)** Show that the tangents intersect at the point $\left(\sqrt{ab}, \tfrac{1}{2}(\sqrt{a} + \sqrt{b})\right)$. [5]

(ii) Given that l_1 and l_2 intersect at a point with integer coordinates, write down a possible pair of values for a and b. [2]

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